

3.6 Prove Theorems About Perpendicular Lines



Before

You found the distance between points in the coordinate plane.

Now

You will find the distance between a point and a line.

Why?

So you can determine lengths in art, as in Example 4.

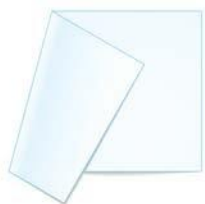
Key Vocabulary

- distance from a point to a line

ACTIVITY FOLD PERPENDICULAR LINES

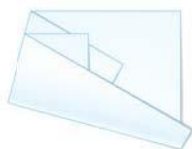
Materials: paper, protractor

STEP 1



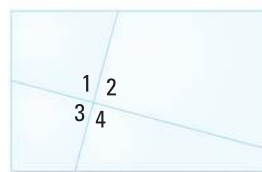
Fold a piece of paper.

STEP 2



Fold the paper again, so that the original fold lines up on itself.

STEP 3



Unfold the paper.

DRAW CONCLUSIONS

- What type of angles appear to be formed where the fold lines intersect?
- Measure the angles with a protractor. Which angles are congruent? Which angles are right angles?

The activity above suggests several properties of perpendicular lines.

THEOREMS

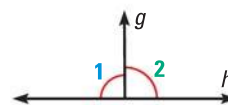
For Your Notebook

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

Proof: Ex. 31, p. 196

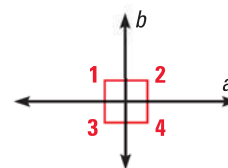


THEOREM 3.9

If two lines are perpendicular, then they intersect to form four right angles.

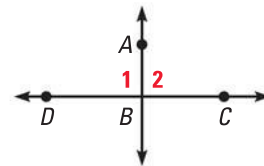
If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are right angles.

Proof: Ex. 32, p. 196



EXAMPLE 1 Draw conclusions

In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$. What can you conclude about $\angle 1$ and $\angle 2$?



Solution

\overleftrightarrow{AB} and \overleftrightarrow{BC} are perpendicular, so by Theorem 3.9, they form four right angles. You can conclude that $\angle 1$ and $\angle 2$ are right angles, so $\angle 1 \cong \angle 2$.

THEOREM

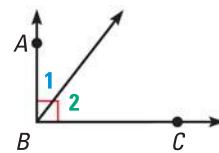
For Your Notebook

THEOREM 3.10

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If $\overleftrightarrow{BA} \perp \overleftrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.

Proof: Example 2, below

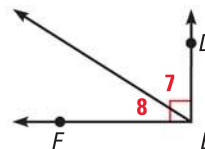


EXAMPLE 2 Prove Theorem 3.10

Prove that if two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

GIVEN $\overleftrightarrow{ED} \perp \overleftrightarrow{EF}$

PROVE $\angle 7$ and $\angle 8$ are complementary.

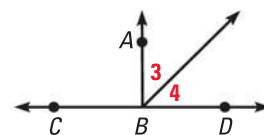


STATEMENTS	REASONS
1. $\overleftrightarrow{ED} \perp \overleftrightarrow{EF}$	1. Given
2. $\angle DEF$ is a right angle.	2. \perp lines intersect to form 4 rt. \angle s. (Theorem 3.9)
3. $m\angle DEF = 90^\circ$	3. Definition of a right angle
4. $m\angle 7 + m\angle 8 = m\angle DEF$	4. Angle Addition Postulate
5. $m\angle 7 + m\angle 8 = 90^\circ$	5. Substitution Property of Equality
6. $\angle 7$ and $\angle 8$ are complementary.	6. Definition of complementary angles



GUIDED PRACTICE for Examples 1 and 2

1. Given that $\angle ABC \cong \angle ABD$, what can you conclude about $\angle 3$ and $\angle 4$? Explain how you know.



2. Write a plan for proof for Theorem 3.9, that if two lines are perpendicular, then they intersect to form four right angles.

THEOREMS

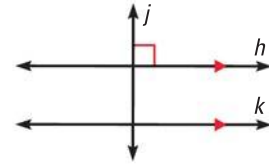
For Your Notebook

THEOREM 3.11 Perpendicular Transversal Theorem

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.

Proof: Ex. 42, p. 160; Ex. 33, p. 196

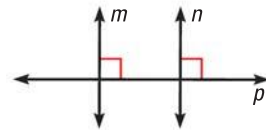


THEOREM 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Proof: Ex. 34, p. 196

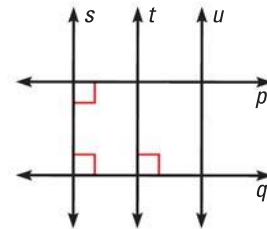


EXAMPLE 3 Draw conclusions

Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

Solution

Lines p and q are both perpendicular to s , so by Theorem 3.12, $p \parallel q$. Also, lines s and t are both perpendicular to q , so by Theorem 3.12, $s \parallel t$.

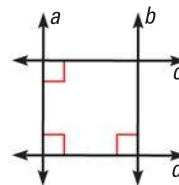


GUIDED PRACTICE for Example 3

Use the diagram at the right.

3. Is $b \parallel a$? Explain your reasoning.

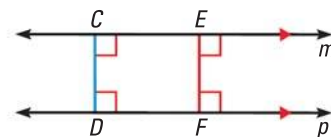
4. Is $b \perp c$? Explain your reasoning.



DISTANCE FROM A LINE The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point A and line k is AB . You will prove this in Chapter 5.



Distance from a point to a line

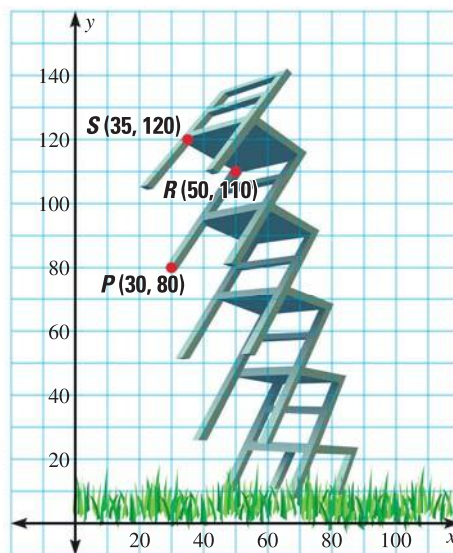


Distance between two parallel lines

The **distance between two parallel lines** is the length of any perpendicular segment joining the two lines. For example, the distance between line p and line m above is CD or EF .

EXAMPLE 4 Find the distance between two parallel lines

SCULPTURE The sculpture below is drawn on a graph where units are measured in inches. What is the approximate length of \overline{SR} , the depth of a seat?

**Solution**

You need to find the length of a perpendicular segment from a back leg to a front leg on one side of the chair.

Using the points $P(30, 80)$ and $R(50, 110)$, the slope of each leg is

$$\frac{110 - 80}{50 - 30} = \frac{30}{20} = \frac{3}{2}.$$

The segment \overline{SR} has a slope of

$$\frac{120 - 110}{35 - 50} = -\frac{10}{15} = -\frac{2}{3}.$$

The segment \overline{SR} is perpendicular to the leg so the distance SR is

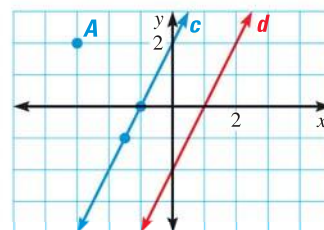
$$d = \sqrt{(35 - 50)^2 + (120 - 110)^2} \approx 18.0 \text{ inches.}$$

► The length of \overline{SR} is about 18.0 inches.

**GUIDED PRACTICE for Example 4**

Use the graph at the right for Exercises 5 and 6.

5. What is the distance from point A to line c ?
6. What is the distance from line c to line d ?



7. Graph the line $y = x + 1$. What point on the line is the shortest distance from the point $(4, 1)$? What is the distance? Round to the nearest tenth.

3.6 EXERCISES

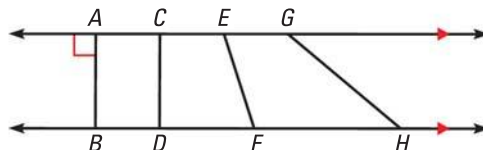
HOMESWORK KEY

○ = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 19, 23, and 29

★ = **STANDARDIZED TEST PRACTICE**
Exs. 11, 12, 21, 22, and 30

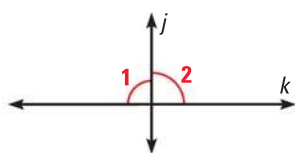
SKILL PRACTICE

1. **VOCABULARY** The length of which segment shown is called the distance between the two parallel lines? *Explain.*

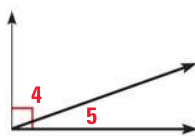


- JUSTIFYING STATEMENTS** Write the theorem that justifies the statement.

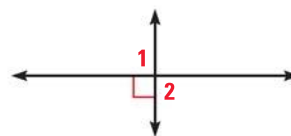
2. $j \perp k$



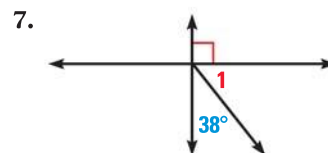
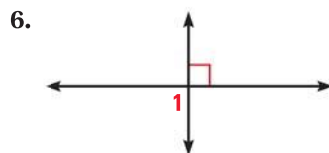
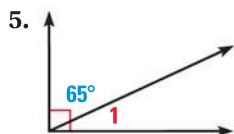
3. $\angle 4$ and $\angle 5$ are complementary.



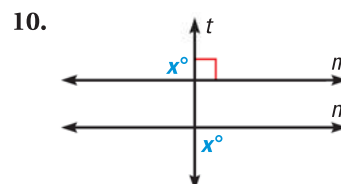
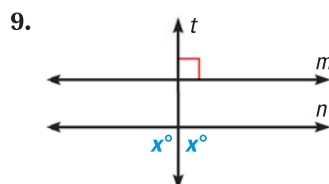
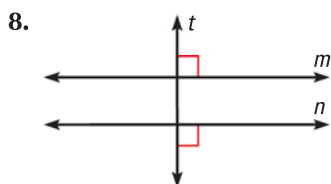
4. $\angle 1$ and $\angle 2$ are right angles.



- APPLYING THEOREMS** Find $m\angle 1$.

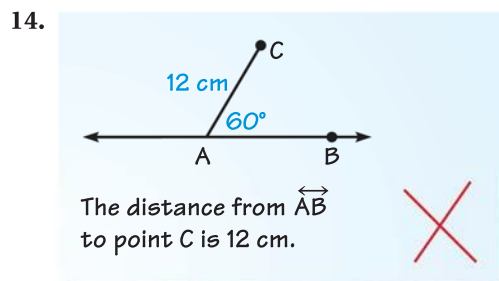
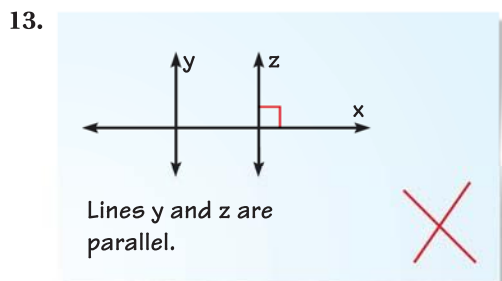


- SHOWING LINES PARALLEL** *Explain how you would show that $m \parallel n$.*



11. ★ **SHORT RESPONSE** *Explain how to draw two parallel lines using only a straightedge and a protractor.*
12. ★ **SHORT RESPONSE** *Describe how you can fold a sheet of paper to create two parallel lines that are perpendicular to the same line.*

- ERROR ANALYSIS** *Explain why the statement about the figure is incorrect.*



EXAMPLES 1 and 2

on p. 191
for Exs. 2–7

EXAMPLE 3

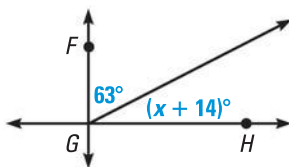
on p. 192
for Exs. 8–12

EXAMPLES 3 and 4

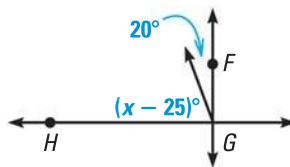
on pp. 192–193
for Exs. 13–14

FINDING ANGLE MEASURES In the diagram, $\overrightarrow{FG} \perp \overrightarrow{GH}$. Find the value of x .

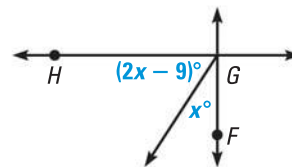
15.



16.

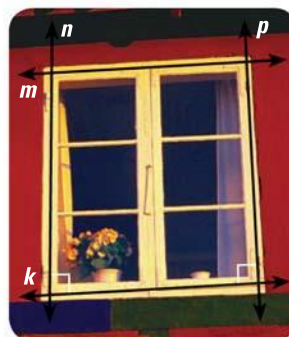


17.

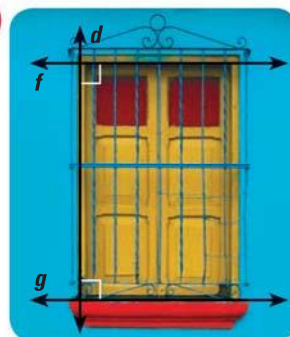


DRAWING CONCLUSIONS Determine which lines, if any, must be parallel. Explain your reasoning.

18.



19.

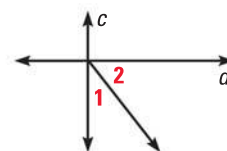


20.



21. **★ MULTIPLE CHOICE** Which statement must be true if $c \perp d$?

- (A) $m\angle 1 + m\angle 2 = 90^\circ$ (B) $m\angle 1 + m\angle 2 < 90^\circ$
 (C) $m\angle 1 + m\angle 2 > 90^\circ$ (D) Cannot be determined



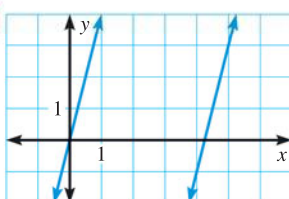
22. **★ WRITING** Explain why the distance between two lines is only defined for parallel lines.

EXAMPLE 4

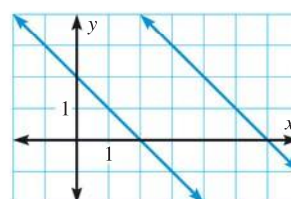
on p. 193
for Exs. 23–24

FINDING DISTANCES Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.

23.



24.

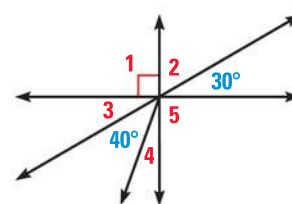


25. **CONSTRUCTION** You are given a line n and a point P not on n . Use a compass to find two points on n equidistant from P . Then use the steps for the construction of a segment bisector (page 33) to construct a line perpendicular to n through P .

26. **FINDING ANGLES** Find all the unknown angle measures in the diagram at the right. Justify your reasoning for each angle measure.

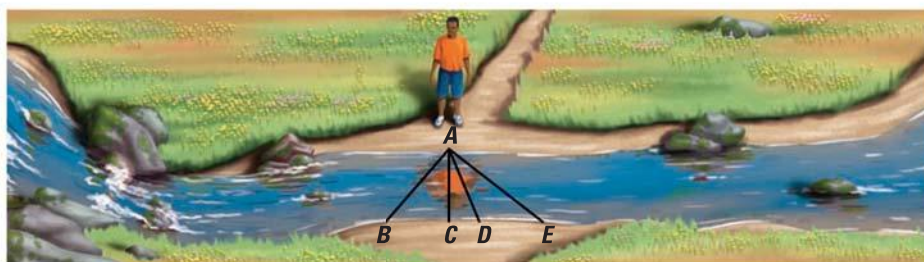
27. **FINDING DISTANCES** Find the distance between the lines with the equations $y = \frac{3}{2}x + 4$ and $-3x + 2y = -1$.

28. **CHALLENGE** Describe how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? Explain.



PROBLEM SOLVING

29. **STREAMS** You are trying to cross a stream from point A. Which point should you jump to in order to jump the shortest distance? *Explain.*



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30. ★ **SHORT RESPONSE** The segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments would look like if they were perpendicular to the crosswalk. Which method requires less paint? *Explain.*

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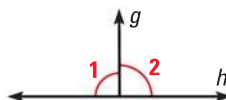


EXAMPLE 2
on p. 191
for Exs. 31–34

31. **PROVING THEOREM 3.8** Copy and complete the proof that if two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

GIVEN ► $\angle 1$ and $\angle 2$ are a linear pair.
 $\angle 1 \cong \angle 2$

PROVE ► $g \perp h$



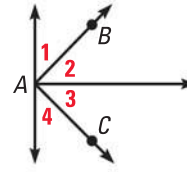
STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are a linear pair.	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary.	2. ?
3. ?	3. Definition of supplementary angles
4. $\angle 1 \cong \angle 2$	4. Given
5. $m\angle 1 = m\angle 2$	5. ?
6. $m\angle 1 + m\angle 1 = 180^\circ$	6. Substitution Property of Equality
7. $2(m\angle 1) = 180^\circ$	7. Combine like terms.
8. $m\angle 1 = 90^\circ$	8. ?
9. ?	9. Definition of a right angle
10. $g \perp h$	10. ?

PROVING THEOREMS Write a proof of the given theorem.

32. Theorem 3.9
33. Theorem 3.11, Perpendicular Transversal Theorem
34. Theorem 3.12, Lines Perpendicular to a Transversal Theorem

CHALLENGE Suppose the given statement is true. Determine whether $\overrightarrow{AB} \perp \overrightarrow{AC}$.

35. $\angle 1$ and $\angle 2$ are congruent.
 36. $\angle 3$ and $\angle 4$ are complementary.
 37. $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$
 38. $m\angle 1 = 40^\circ$ and $m\angle 4 = 50^\circ$

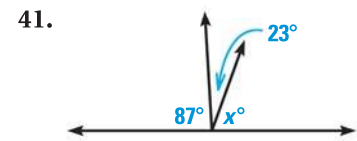
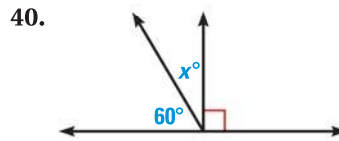
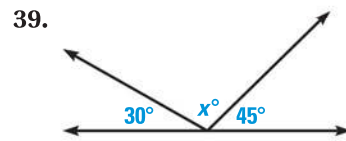


MIXED REVIEW

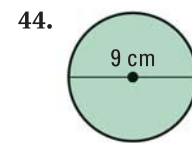
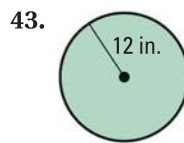
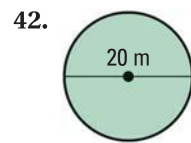
PREVIEW

Prepare for
Lesson 4.1
in Exs. 39–41.

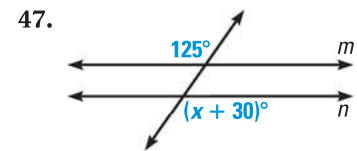
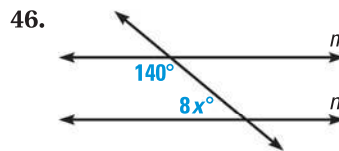
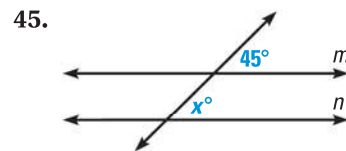
Find the value of x . (p. 24)



Find the circumference and area of the circle. Round to the nearest tenth.
(p. 49)



Find the value of x that makes $m \parallel n$. (p. 161)



QUIZ for Lessons 3.5–3.6

Write an equation of the line that passes through point P and is parallel to the line with the given equation. (p. 180)

1. $P(0, 0)$, $y = -3x + 1$ 2. $P(-5, -6)$, $y - 8 = 2x + 10$ 3. $P(1, -2)$, $x = 15$

Write an equation of the line that passes through point P and is perpendicular to the line with the given equation. (p. 180)

4. $P(3, 4)$, $y = 2x - 1$ 5. $P(2, 5)$, $y = -6$ 6. $P(4, 0)$, $12x + 3y = 9$

Determine which lines, if any, must be parallel. Explain. (p. 190)

